Ph. D Entrance Test – Jan 21 (Mathematics)

1. The number of idempotent elements in a group is

a) Always even
b) Always odd greater than one
c) Always an odd prime
d) Only one

2. The set of integers with operation $*$ defined by $a*b=a+b+1$ is given to be a group. The identity of this group is:
a) 1
b) 0
c) -1
d) 2

3. Let G be a non abelian group. Then its order can be

a) 25

b) 55

c) 65

d) 35

4. Which of the following can be the class equation of a group of order 10?

a) 1+1+1+2+5=10

b) 1+2+3+4=10

c) 1+2+2+5=10

d) 1+1+2+2+2+2=10

5. Let A be a 3×3 matrix whose characteristic roots are 3, 2, -1. If $B=A^2-A$, then $|B|$ is
a) 24
b) -2
c) 12
d) -12

6. The eigen values of the matrix of the quadratic form associated with the quadratic equation $9x^2-4xy+6y^2-10x-20y=5$ are

a) 1, 2
b) 2, 5
c) 5, 10
d) -5, 10

7. Which one of the following form a basis for the vector space $\Re^3$:
a) $(1,1,1)$ and $(1,-1,5)$ (1, 1, 1)
b) $(1,2,3)$, $(1,0,-1)$, $(3,-1,0)$ and $(2,1,-2)$

c) $(1,2,5)$, $(1,1,4)$ and $(2,4,10)$
d) $(1,1,1)$, $(1,2,3)$ and $(2,-1,1)$

8. If S and T are linear operators on $\Re^2$ defined by $S(x,y)=(0,x)$ and $T(x,y)=(x,0)$ then:
a) $ST=0$
b) $TS\neq 0$
c) $S+T=0$
d) $TS=0$

9. Which of the following statements is not true:
a) The set of natural numbers is order-complete
b) The set of integers is order-complete
c) The set of rational numbers is order-complete
d) The set of real numbers is order complete

10. The series $\sum\frac{(-1)^{n+1}}{n^p}$ is
a) absolutely convergent for every p
b) absolutely convergent for $p<1$
c) absolutely convergent for $p=1$
d) absolutely convergent for $p>1$ but conditionally convergent for $0<p\leq 1$

11. The function f defined by $f(x)=\begin{cases}x, x \text{ is irrational}\\ -x, x \text{ is rational}\end{cases}$ is
a) continuous for every real x
b) continuous when x is rational
c) continuous when x is irrational
d) continuous only at $x=0$

12. The sequence $\left\langle\frac{nx}{1+n^3x^2}\right\rangle$ converges uniformly to zero
a) only for $0<x<1$
b) only for $0\leq x<1$
c) only for $0<x\leq 1$
d) for $0\leq x\leq 1$

13. The number $\sqrt{2}e^{i\pi}$ is
a) A rational number
b) An imaginary number
c) An irrational number
d) A Transcendental number

14. The function $f(z)=\log(z^2+z-2)$ has branch points at
a) $z=1, z=-1$

b) $z=2, z=-2$

c) $z=1, z=-2$

d) None of these

15. Critical points of the bilinear transformation $w(z)=\frac{az+b}{cz+d}$ are

a) $z=-\frac{d}{c}, z=0$

b) $z=-\frac{d}{c}, z=\infty$

c) $z=0, z=\infty$

d) $z=-\frac{d}{c}, z=-\frac{b}{a}$

16. If z_1, z_2, z_3, z_4 are distinct points in the order in which they are written, then the number of distinct cross ratios are

a) 4!

b) 4^4

c) 6

d) 1

17. The solution of $\frac{dy}{dx}=y^2$, $y(0)=1$ exists for all

a) $x\in(-\infty,1)$

b) $x\in[0,a]$ where $a>1$

c) $x\in(-\infty,\infty)$

d) $x\in[1,a]$ where $a>1$

18. If e^{-x}, xe^{-x} are solutions of $y''+ay'+by=0$, then

a) $a=0, b=1$

b) $a=1, b=2$

c) $a=2, b=1$

d) $a=-1, b=0$

19. General solution of the partial differential equation $(y^2+z^2-x^2)p-2xyq+2xz=0$ is

a) $\phi\left[\frac{y}{z}, \frac{x^2+y^2+z^2}{z}\right]=0$

b) $\phi\left[\frac{x}{z}, \frac{x^2+y^2+z^2}{y}\right]=0$

c) $\phi\left[\frac{y}{z}, \frac{x^2-y^2+z^2}{z}\right]=0$

d) $\phi\left[\frac{x}{z}, \frac{x^2-y^2+z^2}{z}\right]=0$

20. Let W be the wronskian of two linearly independent solutions of ODE $2y''+y'+t^2y=0; t \in R$. Then, for all t, there exists a constant C ϵR such that W(t) is

a) Ce^{-t}

b) $Ce^{-t/2}$

c) Ce^{2t}

d) Ce^{-2t}

21. Let y: R→R be a solution of the ODE $\frac{d^2y}{dx^2}-y=e^{-x}, x \in R, \ y=\frac{dy}{dx}=0$ when $x=0$

a) Y is bounded on R

b) $\lim_{x\to\infty} e^{-x}y(x)=\frac{1}{4}$

c) $\lim_{x\to\infty} e^{x}y(x)=\frac{1}{4}$

d) None of these

22. The value of constant α in the third order Runga-Kutta method $u_{j+1}=u_j+\frac{1}{8}(2k_1+\alpha k_2+3k_3)$ is

a) 3
b) 4
c) 2
d) 1

23. Which of the following method always converges to root of equation $f(x)=0$?
a) Newton Raphson method
b) Regula False method
c) Secant Method
d) All of Above

24. Every discrete topological space is:
a) First countable
b) Second countable
c) Connected
d) Compact

25. Let $\mathbb{R}$ be a real line with usual topology. Then the space $\mathbb{R} \times \mathbb{R}\backslash\mathbb{Q} \times \mathbb{Q}$ is

a) Path connected

b) not connected

c) Countable

d) connected but not path connected

26. A simple pendulum is suspended with a moving support, the constraints involved are

a) Holonomic

b) Non-holonomic

c) Scleronomic

d) Rheonomic

27. In a two-body equivalent to a single body problem, if μ is the reduced mass of the system with masses m_1 and m_2, then μ is given by

a) $\mu = \dfrac{m_1^2 + m_2^2}{2}$

b) $\mu = \dfrac{m_1 + m_2}{2}$

c) $\mu = \sqrt{m_1 m_2}$

d) None of these

28. Green Theorem gives the relation between

a) Line and double integral

b) Line and surface integral

c) Double and volume integral

d) None of these

29. Fredholm equation of First kind is

a) $y(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt$

b) $f(x) + \lambda \int_a^b k(x,t)y(t)dt = 0$

c) $y(x) = \lambda \int_a^b k(x,t)y(t)dt$

d) None of these

30. In Fourier Transform problems if $u(x,t)_{x=0}$ is given then to remove $\frac{\partial^2 u}{\partial x^2}$ form the equation we use

a) Infinite Cosine Transform

b) Infinite Sine Transform

c) Finite Sine Transform

d) Finite Cosine Transform

31. Laplace transformation of $F(t); t > 0$ is defined as

a) $\int_0^1 e^{-st} f(s)ds$

b) $\int_0^\infty e^{-st} F(t)dt$

c) $\int_{-\infty}^\infty e^{-st} F(t)ds$

d) None of these

32. Which of the following is true regarding the multiplication law of probabilities:

a) $P(A \cap B) = P(A)\ P(B)$
b) $P(A \cap B) = P(B)\ P(B/A)$
c) $P(A \cap B) = P(A)\ P(B/A)$
d) $P(A \cap B) = P(A)\ P(A/B)$

33. Choose the correct statement:

a) A real valued function defined on a sample space and taking real values is called a one-dimensional random variable
b) A real valued function defined on a sample space and taking any values is called a one-dimensional random variable
c) A real valued function defined on a sample space and taking any type of values is called two-dimensional random variable

d) A real valued function defined on a sample space and taking real values is called two-dimensional random variable

34. Choose the correct statement:

a) If χ^2 is a chi-square variate with n degrees of freedom, then $\frac{\chi^2}{2}$ is a beta variate with parameter $\frac{n}{2}$.

b) If χ^2 is a chi-square variate with n degrees of freedom, then $\frac{\chi^2}{2}$ is a beta variate with parameter n.

c) If χ^2 is a chi-square variate with n degrees of freedom, then $\frac{\chi^2}{2}$ is a gamma variate with parameter $\frac{n}{2}$.

d) If χ^2 is a chi-square variate with n degrees of freedom, then $\frac{\chi^2}{2}$ is a gamma variate with parameter n.

35. With the usual notations of systematic sampling, choose the correct relation:

a) $V_S : V_{Sy} : V_R \cong \frac{1}{n} : 1 : n$

b) $V_S : V_{Sy} : V_R \cong n : \frac{1}{n} : 1$

c) $V_S : V_{Sy} : V_R \cong \frac{1}{n} : n : 1$

d) $V_S : V_{Sy} : V_R \cong 1 : \frac{1}{n} : n$

36. Choose the correct statement:

a) The regression estimate is always more efficient than the ratio estimate unless regression of y on x is a straight line passing through the origin.
b) The regression estimate is always more efficient than the ratio estimate unless regression of y on x is not a straight line passing through the origin.
c) The regression estimate is always more efficient than the ratio estimate.
d) The regression estimate is never efficient than the ratio estimate.

37. A statistic $t(x_1, x_2, \ldots, x_n)$ is said to be minimum variance unbiased estimator of a certain parameter θ if

a) $E(t) = \theta$ and $Var(t) > Var(t')$

b) $E(t) = \theta$ and $Var(t) < Var(t')$

c) $E(t) < \theta$ and $Var(t) > Var(t')$

d) $E(t) < \theta$ and $Var(t) < Var(t')$

where t' is any other unbiased estimator of θ

38. Choose the correct statement:

a) Randomized Block diagram (R.B.D.) is not efficient than C.R.D. for most of the experimental work.
b) R.B.D. is suitable for large number of treatments.
c) In R.B.D. no restrictions are placed on the number of treatments.
d) In R.B.D. statistical analysis is complicated.

39. Which of the following is an incorrect statement?

a) Completely randomized design (C.R.D.) is useful in laboratory techniques.
b) Completely randomized design (C.R.D.) is useful where the experimental material is homogeneous.
c) Completely randomized design (C.R.D.) is useful in situations where an appreciable fraction of units is likely to be destroyed.
d) None of the above situations.

40. Which of the following is true regarding poisson process?

a) The distribution of arrivals is always poisson.
b) The distribution of arrivals is always exponential.
c) The distribution of inter arrival times is always poisson.
d) The distribution of arrivals may not follow a particular distribution.

Math

Ph.D Ent. Test 21

Key-I

1. d
2. c
3. b
4. c
5. a
6. c
7. d
8. d
9. c
10. d
11. d
12. d
13. c
14. c
15. b
16. a
17. a
18. c
19. a
20. b
21. b
22. a
23. b
24. a
25. a
26. d
27. d
28. a
29. b
30. b
31. b
32. c
33. a
34. c
35. a
36. a
37. b
38. c
39. d
40. a