MAHARAJA RANJIT SINGH PUNJAB TECHNICAL UNIVERSITY DABWALI ROAD, BATHINDA

[Established by State Government of Punjab vide Act No. 5 of 2015 u/s UGC Act 2(f)]

PET-2017 (Math– Faculty of Sciences)

Roll No: ..... Date... 25 JUN 2017

Signature of the Candidate: .....

1. Let $f : X \to Y$ and $g : Y \to Z$ be maps. Which of the following statement is correct?

   (i) If $g \circ f$ is surjective, then g is surjective.
   (ii) If $g \circ f$ is injective, then f is injective.
   (iii) If $g \circ f$ is bijective, then f and g are bijective.
   (iv) All are correct

   a) (i) and (ii)
   b) (i), (ii) and (iii)
   c) only (iii)
   d) only (iv)

2. If G is a group of even order, then its elements of order two are

   a) At least one
   b) At least two
   c) Exactly two
   d) At most two

3. A group of order $11^2.12^2$ has:

   a) Only one 11-Sylow subgroup
   b) Two 11-Sylow subgroups
   c) Eleven 11-Sylow subgroups
   d) Thirteen 11-Sylow subgroups

4. The number of conjugacy classes in the permutation group S_5 is

   a) 12
   b) 11
   c) 10
   d) 7

MAHARAJA RANJIT SINGH PUNJAB TECHNICAL UNIVERSITY BATHINDA

25 JUN 2017

5. Let A be a matrix of order $m \times n$ and B be a matrix of order $n \times p$, $n > p$. If rank $(A) = n$ and rank $(B) = p$, then the rank (AB) is

a) n
b) p
c) np
d) $n + p$

6. If α is a characteristic root of a non-singular matrix, then characteristic root of $Adj.\ A$ is

a) $\alpha |A|$
b) α
c) $\frac{|A|}{\alpha}$
d) $\frac{|Adj.A|}{\alpha}$

7. If T be a linear operator on a vector space V such that $T^2 - T + I = 0$, then

a) T is 1-1 but may not be onto
b) T is onto but may not be one-one
c) T is invertible
d) No such T exists

8. Which one of the following mapping is linear:

a) $T : \Re^2 \to \Re$ defined by $T(x, y) = xy$
b) $T : \Re^2 \to \Re$ defined by $T(x, y) = (x + 1, 2y, x + y)$
c) $T : \Re^2 \to \Re$ defined by $T(x, y) = (x + y, x)$
d) $T : \Re^2 \to \Re$ defined by $T(x, y) = (|x| = 0)$

9. Let $f_n(x) = \begin{cases} 1 - nx, & x \in \left[0, \frac{1}{n}\right] \\ 0, & x \in \left[\frac{1}{n}, 1\right] \end{cases}$, then

a) $\lim_{n \to \infty} f_n(x)$ defines a continuous function on $[0,1]$
b) $f_n(x)$ converges uniformly on $[0,1]$
c) $\lim_{n \to \infty} f_n(x) = 0$ for all $x \in [0,1]$
d) $\lim_{n \to \infty} f_n(x)$ exists for all $x \in [0,1]$

10. If $a_n = (-1)^n \left\{1 + \frac{1}{n}\right\}, n \in N$, then

a) $\underline{\lim} a_n = 0$

b) $\underline{\lim} a_n = 1$

c) $\underline{\lim} a_n = -1$

d) $\overline{\lim} a_n = 0$

11. If $f(x) = \frac{|x|}{[x]}$, $x \notin [0,1]$ then $\lim_{x \to 2+} \frac{f(x) - f(2)}{x - 2}$ equals

a) 1

b) $\frac{1}{2}$

c) $\frac{1}{4}$

d) does not exist

12. The improper integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^p} dx$ converges for

a) $p > 2$

b) $p \leq 2$

c) $p < 2$

d) $0 < p < 40$

13. The function $f(z) = \tan z$

a) has poles at $z = \frac{(2n+1)\pi}{2}$, $n \in Z$

b) is an entire function

c) has no zeros in C

d) has a removable singularity at $z = 0$

14. The cross ratio of the four points (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a

a) Circle

b) Straight line

c) Circle and on a straight line

d) Circle or on a straight line

15. The points which coincide with their transformations are called

a) Fixed points

b) Critical points

c) Bilinear points

d) Conformal points

16. For the function $\frac{\sin z}{z}$, the point $z = 0$ is:

a) an isolated singularity
b) a pole
c) a removable singularity
d) an essential singularity

17. The solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$ is

a) $y = c_1 x + c_2 x^2 + \frac{x}{2}$

b) $y = c_1 x + c_2 x^3 + \frac{x^2}{2}$

c) $y = c_1 x^2 + c_2 x^3 + \frac{x}{2}$

d) $y = c_1 x^2 + c_2 x^3 + \frac{x^2}{2}$

18. The general solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

a) $u = f(x+iy) + g(x-iy)$
b) $u = f(x+y) + g(x-y)$
c) $u = cf(x-iy)$
d) $u = g(x+y)$

19. The partial differential equation for $2z = \frac{x^2}{a} + \frac{y^2}{b}$ is

a) $2z = x\frac{\partial z}{\partial x}$

b) $2z = y\frac{\partial z}{\partial y}$

c) $z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

d) $2z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

20. The singular integral of ODE $(xy' - y)^2 = x^2(x^2 - y^2)$ is

a) $y = x\sin x$

b) $y = x\sin\left(x + \frac{\pi}{4}\right)$

c) $y = x$

d) $y = x + \frac{\pi}{4}$

25 JUN 2017

21. Let $a, b \in R$ be such that $a^2 + b^2 \neq 0$. Then the Cauchy Problem

$$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = 1;\ x, y \in R\ u(x, y) = x \text{ on } ax + by = 1$$

a) has more than one solution if either a or b is 0

b) has no solution

c) has a unique solution

d) has infinitely many solutions

22. The condition for convergence of the Newton Raphson method to a root α is

a) $\frac{f'(\alpha)}{2f''(\alpha)} < 1$

b) $\frac{f'(\alpha)}{f''(\alpha)} < 1$

c) $\frac{f'(\alpha)}{f''(\alpha)} > 1$

d) None of these

23. If the value of $f(x)$ given only at $x = 0, \frac{1}{3}, \frac{2}{3}, 1$, then which of the following can be used to evaluate $\int_0^1 f(x)dx$ approximately.

a) Trapezoidal rule

b) Simpson rule

c) Trapezoidal as well as Simpson's rule

d) None of these

24. A subset A of a topological space X is open as well as closed if and only if:

a) Frontier $A \neq \phi$

b) Frontier $A = \phi$

c) Frontier $A = A$

d) Frontier $A \neq A$

25. Let $S = [0,1) \cup [2,3]$ and $f : S \to \square$ be a strictly monotone increasing function such that $f(S)$ is connected. Which of the following is true?

a) f has exactly one discontinuity

b) f has exactly two discontinuities

c) f has infinitely many discontinuities

d) f is continuous

25 JUN 2017

26. Hamilton's principle states that the actual motion of a system takes place in such a manner that the time integral of _____ function taken between two configurations of the system has _____ value.

a) Lagrangian, an extremum
b) Lagrangian, any constant
c) Hamiltonian, an extremum
d) Hamiltonian, any constant

27. The curve lying on the surface of a right circular cylinder along which the distance between any two points is minimum is

a) Cycloid
b) Spiral
c) Circular Helix
d) Catenary

28. Stoke's theorem gives the relation between

a) Line and double integral
b) Line and surface integral
c) Double and volume integral
d) None of these

29. Fredholm equation of Second kind is

a) $y(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt$

b) $f(x) + \lambda \int_a^b k(x,t)y(t)dt = 0$

c) $y(x) = \lambda \int_a^b k(x,t)y(t)dt$

d) None of these

30. In Fourier Transform problems if $[\frac{\partial u}{\partial x}]_{x=0}$ is given then to remove $\frac{\partial^2 u}{\partial x^2}$ form the equation we use

a) Infinite Cosine Transform
b) Infinite Sine Transform
c) Finite Sine Transform
d) Finite Cosine Transform

25 JUN 2017

31. Laplace transformation of is $e^t \sin t$ is

a) $\dfrac{s+1}{s^2+2s+2}$

b) $\dfrac{s-1}{s^2-2s+2}$

c) $\dfrac{s-1}{s^2-2s-2}$

d) None of these

32. By Bayes theorem, which of the following probabilities are calculated:

a) Priori probabilities
b) Likelihood probabilities
c) Posteriori probabilities
d) Conditional probabilities

33. If χ_1^2 and χ_2^2 are two independent χ^2-variates with n_1 and n_2 degrees of freedom respectively, then which of the following relations is true:

a) $\dfrac{\chi_1^2}{\chi_2^2} \sim \beta_1\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$

b) $\dfrac{\chi_1^2}{\chi_2^2} \sim \beta_2\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$

c) $\dfrac{\chi_1^2}{\chi_1^2+\chi_2^2} \sim \beta_2\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$

d) $\dfrac{\chi_1^2}{\chi_1^2+\chi_2^2} \sim \chi_n^2$

34. If $F(n_1, n_2)$ denotes F-distribution with (n_1, n_2) degrees of freedom, then which of the following is a correct statement:

a) As $n_2 \to \infty$, $\chi^2 = n_1 F$ follows χ^2-distribution with $(n_1 - 1)$ degrees of freedom.
b) As $n_2 \to \infty$, $\chi^2 = n_1 F$ follows χ^2-distribution with n_1 degrees of freedom.
c) As $n_1 \to \infty$, $\chi^2 = n_2 F$ follows χ^2-distribution with n_2 degrees of freedom.
d) As $n_1 \to \infty$, $\chi^2 = n_2 F$ follows χ^2-distribution with $(n_2 - 1)$ degrees of freedom.

35. If s^2 and S^2 denote sample mean square and population mean square respectively, then the variance of the sample mean $\bar{y}_n$ of the simple random sampling is given by

a) $V(\bar{y}_n) = \dfrac{N-n}{N} \cdot \dfrac{S}{n}$ b) $V(\bar{y}_n) = \dfrac{1}{N} \cdot \dfrac{S^2}{n}$ c) $V(\bar{y}_n) = \dfrac{1}{N} \cdot \dfrac{S^2}{\sqrt{n}}$

d) $V(\bar{y}_n) = \dfrac{N-n}{N} \cdot \dfrac{S^2}{n}$

25 JUN 2017

36. Choose the incorrect statement:

a) The sampling procedure in which the units are selected randomly is known as pps sampling.

b) The sampling procedure in which the units are selected with probabilities proportional to some measure of their size is known as pps sampling.

c) When the sampling unit is a cluster, the procedure of sampling is called cluster sampling.

d) Stratified sampling is important because of administrative reasons.

37. If $\hat{\theta}_1$ and $\hat{\theta}_2$ denotes consistent estimators of a certain parameter θ and $V\left(\hat{\theta}_1\right)$ and $V\left(\hat{\theta}_2\right)$ denote their respective variances, then $\hat{\theta}_1$ is more efficient than $\hat{\theta}_2$ if

a) $V\left(\hat{\theta}_1\right)=V\left(\hat{\theta}_2\right)$

b) $V\left(\hat{\theta}_1\right)>V\left(\hat{\theta}_2\right)$

c) $V\left(\hat{\theta}_1\right)<V\left(\hat{\theta}_2\right)$

d) $V\left(\hat{\theta}_1\right)=\frac{1}{2}V\left(\hat{\theta}_2\right)$

38. Choose the incorrect statement:

a) Latin Square Design (L.S.D.) is an incomplete 3-way layout.

b) In L.S.D. more than one factor can be investigated simultaneously.

c) In L.S.D. the number of treatments is restricted to the number of replications.

d) In the field layout, R.B.D. is much complicated to manage than L.S.D.

39. Choose the incorrect statement:

a) A system is said to be in transient state when its operating characteristics are dependent on time.

b) A system is said to be in transient state when its operating characteristics are not dependent on time.

c) A system is said to be in steady state when its operating characteristics are independent of time.

d) If the arrival rate of the system is more than the servicing rate, the state of the system is known as explosive state.

40. Which of the following is true regarding birth death process?

a) Arrivals can be considered as births and departures as deaths to the system.

b) Arrivals can be considered as deaths and departures as births to the system.

c) Arrivals and departures can be considered as births to the system.

d) Arrivals and departures can be considered as deaths to the system.